BOUNDS ON ANOMALOUS GAUGE BOSON COUPLINGS FROM PARTIAL Z WIDTHS AT LEP

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Abstract

We place bounds on anomalous gauge boson couplings from LEP data with particular emphasis on those couplings which do not contribute to Z decays at tree level. We use an effective field theory formalism to compute the one-loop corrections to the $Z \rightarrow \overline{f}f$ decay widths resulting from non-standard model three and four gauge boson vertices. We find that the precise measurements at LEP constrain the three gauge boson couplings at a level comparable to that obtainable at LEPII and LHC.

1 Introduction

High precision measurements at the Z pole at LEP combined with polarized forward backward asymmetries at SLC and other measurements of electroweak observables at lower energies have been used to place stringent limits on new physics beyond the standard model [1, 2].

Under the assumption that the dominant effects of the new physics would show up as corrections to the gauge boson self-energies, the LEP measurements have been used to parameterize the possible new physics in terms of three observables S, T, U [3]; or equivalently ϵ_1, ϵ_2, ϵ_3 [4]. The difference between the two parameterizations is the reference point which corresponds to the standard model predictions. A fourth observable corresponding to the partial width $Z \to b\overline{b}$ has been analyzed in terms of the parameter δ_{bb} [2] or ϵ_b [4].

In view of the extraordinary agreement between the standard model predictions and the observations, it seems reasonable to assume that the $SU(2)_L \times U(1)_Y$ gauge theory of electroweak interactions is essentially correct, and that the only sector of the theory lacking experimental support is the symmetry breaking sector. There are many extensions of the minimal standard model that incorporate different symmetry breaking possibilities. One large class of models is that in which the interactions responsible for the symmetry breaking are strongly coupled. For this class of models one expects that there will be no new particles with masses below 1 TeV or so, and that their effects would show up in experiments as deviations from the minimal standard model couplings.

In this paper we use the latest measurements of partial decay widths of the Z boson to place bounds on anomalous gauge boson couplings. Our paper is organized as follows. In Section 2 we discuss our formalism and the assumptions that go into the relations between the partial widths of the Z boson and the anomalous couplings. In Section 3 we present our results. Finally, in Section 4 we discuss the difference between our calculation and others that can be found in the literature, and assess the significance of our results by comparing them to other existing limits. Detailed analytical formulae for our results are relegated to an appendix.

2 Formalism

We assume that the electroweak interactions are given by an $SU(2)_L \times U(1)_Y$ gauge theory with spontaneous symmetry breaking to $U(1)_{EM}$, and that we do not have any information on the symmetry breaking sector except that it is strongly interacting and that any new particles have masses higher than several hundred GeV. It is well known that this scenario can be described with an effective Lagrangian with operators organized according to the number of derivatives or gauge fields they have. If we call Λ the scale at which the symmetry breaking physics comes in, this organization of operators corresponds to an expansion of amplitudes in powers of (E^2 or v^2)$/\Lambda^2$. For energies $E \leq v$ this is just an expansion in powers of the coupling constant g or g',

and for energies $E \geq v$ it becomes an energy expansion. The lowest order effective Lagrangian for the symmetry breaking sector of the theory is [5]:

$$\mathcal{L}^{(2)} = \frac{v^2}{4} \text{Tr}\Big[D^\mu \Sigma^\dagger D_\mu \Sigma\Big]. \tag{1}$$

In our notation W_μ and B_μ are the $SU(2)_L$ and $U(1)_Y$ gauge fields with $W_\mu \equiv W_\mu^i \tau_i$.[2] The matrix $\Sigma \equiv \exp(i\vec{\omega} \cdot \vec{\tau}/v)$, contains the would-be Goldstone bosons ω_i that give the W and Z their masses via the Higgs mechanism and the $SU(2)_L \times U(1)_Y$ covariant derivative is given by:

$$D_\mu \Sigma = \partial_\mu \Sigma + \frac{i}{2} g W_\mu^i \tau^i \Sigma - \frac{i}{2} g' B_\mu \Sigma \tau_3. \tag{2}$$

Eq. 1 is thus the $SU(2)_L \times U(1)_Y$ gauge invariant mass term for the W and Z. The physical masses are obtained with $v \approx 246$ GeV. This non-renormalizable Lagrangian is interpreted as an effective field theory, valid below some scale $\Lambda \leq 3$ TeV. The lowest order interactions between the gauge bosons and fermions, as well as the kinetic energy terms for all fields, are the same as those in the minimal standard model.

For LEP observables, the operators that can appear at tree-level are those that modify the gauge boson self-energies. To order $\mathcal{O}(1/\Lambda^2)$ there are only three [5, 6]:

$$\mathcal{L}^{(2GB)} = \beta_1 \frac{v^2}{4}\Big(\text{Tr}\Big[\tau_3 \Sigma^\dagger D_\mu \Sigma\Big]\Big)^2 + \alpha_8 g^2 \Big(\text{Tr}\Big[\Sigma \tau_3 \Sigma^\dagger W_{\mu\nu}\Big]\Big)^2 + gg' \frac{v^2}{\Lambda^2} L_{10} \text{Tr}\Big[\Sigma B^{\mu\nu} \Sigma^\dagger W_{\mu\nu}\Big], \tag{3}$$

which contribute respectively to T, U and S. Notice that for the two operators that break the custodial $SU(2)_C$ symmetry we have used the notation of Ref. [5, 6].

In this paper we will consider operators that affect the Z partial widths at the one-loop level. We will restrict our study to only those operators that appear at order $\mathcal{O}(1/\Lambda^2)$ in the gauge-boson sector and that respect the custodial symmetry in the limit $g' \to 0$. They are:

$$
\begin{aligned}
\mathcal{L}^{(4)} &= \frac{v^2}{\Lambda^2}\Big\{ L_1 \Big(\text{Tr}\Big[D^\mu \Sigma^\dagger D_\mu \Sigma\Big]\Big)^2 + L_2 \Big(\text{Tr}\Big[D_\mu \Sigma^\dagger D_\nu \Sigma\Big]\Big)^2 \\
&\quad -ig L_{9L} \text{Tr}\Big[W^{\mu\nu} D_\mu \Sigma D_\nu \Sigma^\dagger\Big] - ig' L_{9R} \text{Tr}\Big[B^{\mu\nu} D_\mu \Sigma^\dagger D_\nu \Sigma\Big]\Big\},
\end{aligned} \tag{4}
$$

where the field strength tensors are given by:

$$
\begin{aligned}
W_{\mu\nu} &= \frac{1}{2}\Big(\partial_\mu W_\nu - \partial_\nu W_\mu + \frac{i}{2} g[W_\mu, W_\nu]\Big) \\
B_{\mu\nu} &= \frac{1}{2}\Big(\partial_\mu B_\nu - \partial_\nu B_\mu\Big)\tau_3.
\end{aligned} \tag{5}
$$

Although this is not a complete list of all the operators that can arise at this order, we will be able to present a consistent picture in the sense that our calculation will

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[2]$Tr(\tau_i \tau_j) = 2\delta_{ij}$.

not require additional counterterms to render the one-loop results finite. Our choice of this subset of operators is motivated by the theoretical prejudice that violation of custodial symmetry must be "small" in some sense in the full theory [7]. We want to restrict our attention to a small subset of all the operators that appear at this order because there are only a few observables that have been measured.

The operators in Eq. 3 and Eq. 4 would arise when considering the effects of those in Eq. 1 at the one-loop level, or from the new physics responsible for symmetry breaking at a scale Λ at order $1/\Lambda^2$. We, therefore, explicitly introduce the factor v^2/Λ^2 in our definition of $\mathcal{L}^{(4)}$ so that the coefficients L_i are naturally of $\mathcal{O}(1)$.[3]

The anomalous couplings that we consider would have tree-level effects on some observables that can be studied in future colliders. They have been studied at length in the literature [8]. In the present paper we will compute their contribution to the Z partial widths that are measured at LEP. These operators contribute to the Z partial widths at the one-loop level. Since we are dealing with a non-renormalizable effective Lagrangian, we will interpret our one-loop results in the usual way of an effective field theory.

We will first perform a complete calculation to order $\mathcal{O}(1/\Lambda^2)$. That is, we will include the one-loop contributions from the operator in Eq. 1 (and gauge boson kinetic energies). The divergences generated in this calculation are absorbed by renormalization of the couplings in Eq. 3. This calculation will illustrate our method, and as an example, we use it to place bounds on L_{10}.

We will then place bounds on the couplings of Eq. 4 by considering their one-loop effects. The divergences generated in this one-loop calculation would be removed in general by renormalization of the couplings in the $\mathcal{O}(1/\Lambda^4)$ Lagrangian of those operators that modify the gauge boson self-energies at tree-level; and perhaps by additional renormalization of the couplings in Eq. 3. This would occur in a manner analogous to our $\mathcal{O}(1/\Lambda^2)$ calculation. Interestingly, we find that we can obtain a completely finite result for the $Z \to \overline{f}f$ partial widths using only the operators in Eq. 3 as counterterms.

However, our interest is to place bounds on the couplings of Eq. 4 so we proceed as follows. We first regularize the integrals in n space-time dimensions and remove all the poles in $n-4$ as well as the finite analytic terms by a suitable definition of the renormalized couplings. We then base our analysis on the leading non-analytic terms proportional to $L_i \log \mu$. These terms determine the running of the $1/\Lambda^4$ couplings and cannot be generated by tree-level terms at that order. It has been argued in the literature [9], that with a carefully chosen renormalization scale μ (in such a way that the logarithm is of order one), these terms give us the correct order of magnitude for the size of the $1/\Lambda^4$ coefficients. We thus choose some value for the renormalization scale between the Z mass and Λ and require that this logarithmic contribution to the renormalized couplings falls in the experimentally allowed range. Clearly, the LEP

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[3] We do not introduce this factor in the $SU(2)_C$ violating couplings β_1 and α_8 since we do not concern ourselves with them in this paper. They are simply used as counterterms for our one-loop calculation.

observables do not measure the couplings in Eq. 4, and it is only from naturalness arguments like the one above, that we can place bounds on the anomalous gauge-boson couplings. From this perspective, it is clear that these bounds are not a substitute for direct measurements in future high energy machines. They should, however, give us an indication for the level at which we can expect something new to show up in those future machines.

We will perform our calculations in unitary gauge, so we set $\Sigma = 1$ in Eqs. 1, 3 and 4. This results in interactions involving three, and four gauge boson couplings, some of which we present in Appendix A. Those coming from Eq. 1 are equivalent to those in the minimal standard model with an infinitely heavy Higgs boson, and those coming from Eq. 4 correspond to the "anomalous" couplings.

For the lowest order operators we use the conventional input parameters: G_F as measured in muon decay; the physical Z mass: M_Z; and $\alpha(M_Z) = 1/128.8$. Other lowest order parameters are derived quantities and we adopt one of the usual definitions for the mixing angle:

$$s_Z^2 c_Z^2 \equiv \frac{\pi \alpha(M_Z)}{\sqrt{2} G_F M_Z^2}. \tag{6}$$

We neglect the mass and momentum of the external fermions compared to the Z mass. In particular, we do not include the b-quark mass since it would simply introduce corrections of order 5% and our results are only order of magnitude estimates. The only fermion mass that is kept in our calculation is the mass of the top-quark when it appears as an intermediate state.

With this formalism we proceed to compute the $Z \to f\overline{f}$ partial width from the following ingredients.

- The $Z \to f\overline{f}$ vertex, which we write as:

$$i\Gamma_\mu = -i\frac{e}{4s_Z c_Z}\gamma_\mu\left[(r_f + \delta r_f)(1 + \gamma_5) + (l_f + \delta l_f)(1 - \gamma_5)\right] \tag{7}$$

  where $r_f = -2Q_f s_Z^2$ and $l_f = r_f + T_{3f}$. The terms δl_f and δr_f occur at one-loop both at order $1/\Lambda^2$ and at order $1/\Lambda^4$ and are given in Appendix B.

- The renormalization of the lowest order input parameters. At order $1/\Lambda^2$ it is induced by tree-level anomalous couplings and one-loop diagrams with lowest order vertices. At order $1/\Lambda^4$ it is induced by one-loop diagrams with an anomalous coupling in one vertex. We present analytic formulae for the self-energies, vertex corrections and boxes in Appendix B. The changes induced in the lowest order input parameters are:

$$
\begin{aligned}
\frac{\Delta\alpha}{\alpha} &= \frac{A_{\gamma\gamma}(q^2)}{q^2}\Big|_{q^2=0} \\
\frac{\Delta M_Z^2}{M_Z^2} &= \frac{A_{ZZ}(M_Z^2)}{M_Z^2} \\
\frac{\Delta G_F}{G_F} &= \frac{2\Gamma_{We\nu}}{\mathcal{A}_0} - \frac{A_{WW}(0)}{M_W^2} + (Z_f - 1) + B_{\text{box}}
\end{aligned}
\tag{8}
$$

4

The self-energies A_{VV} receive tree-level contributions from the operators with L_{10}, β_1 and α_8. They also receive one-loop contributions from the lowest order Lagrangian Eq. 1 and from the operators with L_1, L_2, L_{9L} and L_{9R}. The effective $We\nu$ vertex, $\Gamma_{We\nu}$, receives one-loop contributions from all the operators in Eq. 4. The fermion wave function renormalization factors Z_f and the box contribution to $\mu \rightarrow e\nu\overline{\nu}$, B_{box}, are due only to one-loop effects from the lowest order effective Lagrangian and are thus independent of the anomalous couplings. Notice that B_{box} enters the renormalization of G_F because we work in unitary gauge where box diagrams also contain divergences.

- Tree-level and one-loop contributions to γZ mixing. Instead of diagonalizing the neutral gauge boson sector, we include this mixing as an additional contribution to δl_f and δr_f in Eq. 7:

$$\delta l'_f = \delta r'_f = -\frac{c_Z}{s_Z} r_f \frac{A_{\gamma Z}(M_Z^2)}{M_Z^2} \tag{9}$$

- Wave function renormalization. For the external fermions we include it as additional contributions to δl_f and δr_f as shown in Appendix B. For the Z we include it explicitly.

With all these ingredients we can collect the results from Appendix B into our final expression for the physical partial width. We find:

$$\begin{aligned}
\Gamma(Z \rightarrow f\overline{f}) &= \Gamma_0 Z_Z \Big[1 - \frac{\Delta G_f}{G_f} - \frac{\Delta M_Z^2}{M_Z^2} + \frac{2(l_f \delta l_f + r_f \delta r_f)}{l_f^2 + r_f^2} \\
&\quad - \frac{2r_f(l_f + r_f)}{l_f^2 + r_f^2} \frac{c_Z^2}{s_Z^2 - c_Z^2} \Big(\frac{\Delta G_f}{G_f} + \frac{\Delta M_Z^2}{M_Z^2} - \frac{\Delta \alpha}{\alpha} \Big) \Big],
\end{aligned} \tag{10}$$

where Γ_0 is the lowest order tree level result,

$$\Gamma_0(Z \rightarrow f\overline{f}) = N_{cf}(l_f^2 + r_f^2) \frac{G_F M_Z^3}{12\pi\sqrt{2}}, \tag{11}$$

and N_{cf} is 3 for quarks and 1 for leptons. We write the contributions of the different anomalous couplings to the Z partial widths in the form:

$$\Gamma(Z \rightarrow f\overline{f}) \equiv \Gamma_{SM}(Z \rightarrow f\overline{f}) \Big(1 + \frac{\delta \Gamma_f^{L_i}}{\Gamma_0(Z \rightarrow f\overline{f})} \Big). \tag{12}$$

We use this form because we want to place bounds on the anomalous couplings by comparing the measured widths with the one-loop standard model prediction Γ_{SM}. Using Eq. 12 we introduce additional terms proportional to products of standard model one-loop corrections and corrections due to anomalous couplings. These are small effects that do not affect our results.

We will not attempt to obtain a global fit to the parameters in our formalism from all possible observables. Instead we use the partial Z widths. We believe this approach to be adequate given the fact that the results rely on naturalness assumptions. Specifically we consider the observables:

$$
\begin{aligned}
\Gamma_e &= 83.98 \pm 0.18 \text{ MeV} \quad \text{Ref. [1]} \\
\Gamma_\nu &= 499.8 \pm 3.5 \text{ MeV} \quad \text{Ref. [10]} \\
\Gamma_Z &= 2497.4 \pm 3.8 \text{ MeV} \quad \text{Ref. [10]} \\
R_h &= 20.795 \pm 0.040 \quad \text{Ref. [10]} \\
R_b &= 0.2202 \pm 0.0020 \quad \text{Ref. [10]}
\end{aligned}
\tag{13}
$$

The bounds on new physics are obtained by subtracting the standard model predictions at one-loop from the measured partial widths as in Eq. 12. We use the numbers of Langacker [2] which use the global best fit values for M_t and α_s with M_H in the range $60 - 1000$ GeV. The first error is from the uncertainty in M_Z and Δr, the second is from M_t and M_H, and the one in brackets is from the uncertainty in α_s.

$$
\begin{aligned}
\Gamma_e &= 83.87 \pm 0.02 \pm 0.10 \text{ MeV} \quad \text{Ref. [11]} \\
\Gamma_\nu &= 501.9 \pm 0.1 \pm 0.9 \text{ MeV} \quad \text{Ref. [12]} \\
\Gamma_Z &= 2496 \pm 1 \pm 3 \pm [3] \text{ MeV} \quad \text{Ref. [2]} \\
R_h &= 20.782 \pm 0.006 \pm 0.004 \pm [0.03] \quad \text{Ref. [2]} \\
\delta_{bb}^{new} &= 0.022 \pm 0.011 \quad \text{Ref. [11]}
\end{aligned}
\tag{14}
$$

where $\delta_{bb}^{new} \equiv [\Gamma(Z \to b\bar{b}) - \Gamma(Z \to b\bar{b})^{(SM)}]/\Gamma(Z \to b\bar{b})^{(SM)}$. We add all errors in quadrature.

3 Results

In this section we compute the corrections to the $Z \to f\bar{f}$ partial widths from the couplings of Eq. 4, and compare them to recent values measured at LEP. We treat each coupling constant independently, and compute only its *lowest* order contribution to the decay widths. We first present the complete $\mathcal{O}(1/\Lambda^2)$ results. They illustrate our method and serve as a check of our calculation. We then look at the effect of the couplings $L_{1,2}$ which affect only the gauge-boson self-energies. We then study the more complicated case of the couplings $L_{9L,9R}$. Finally we isolate the non-universal effects proportional to M_t^2. As explained in the previous section, we do not include in our analysis the operators that appear at $\mathcal{O}(1/\Lambda^2)$ that break the custodial symmetry. As long as one is interested in bounding the anomalous couplings one at a time, it is straightforward to include these operators. For example, we discussed the parity violating one in Ref. [14].

3.1   Bounds on L_{10} at order $1/\Lambda^2$.

The operators in Eq. 3 are the only ones that induce a tree-level correction to the gauge boson self-energies to order $\mathcal{O}(1/\Lambda^2)$. This can be seen most easily by working in a physical basis in which the neutral gauge boson self-energies are diagonalized to order $\mathcal{O}(1/\Lambda^2)$. This is accomplished with renormalizations described in the literature [15, 16], and results in modifications to the $W\overline{f}'f$ and the $Zf\overline{f}$ couplings. This tree-level effect on the $Z \to f\overline{f}$ partial width is, of course, well known. It corresponds, *at leading order*, to the new physics contributions to S, T, U or $\epsilon_{1,2,3}$ discussed in the literature [17].

   In this section we do not perform the diagonalization mentioned above, but rather work in the original basis for the fields. This will serve two purposes. It will allow us to present a complete $\mathcal{O}(1/\Lambda^2)$ calculation as an illustration of the method we use to bound the other couplings. Also, because the gauge boson interactions that appear at this order have the same tensor structure as those induced by L_{9L} and L_{9R}, we will be able to carry out the calculation involving those two couplings simultaneously. In this way, even though the terms with $L_{9L,9R}$ are order $1/\Lambda^4$, the calculation to order $1/\Lambda^2$ will serve as a check of our answer for $L_{9L,9R}$.

   To recover the $1/\Lambda^2$ result we set $L_{9L,9R} = 0$ (and also $L_{1,2} = 0$ but these terms are clearly different) in the results of Appendix B. As explained in Section 2, we have regularized our one-loop integrals in n dimensions and isolated the ultraviolet poles $1/\epsilon = 2/(4-n)$. We find that we obtain a finite answer to order $1/\Lambda^2$ if we adopt the following renormalization scheme:[4] [5]

$$\frac{v^2}{\Lambda^2}L_{10}^r(\mu) = \frac{v^2}{\Lambda^2}L_{10} - \frac{1}{16\pi^2}\frac{1}{12}\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{M_Z^2}\right)$$

$$\beta_1^r(\mu) = \beta_1 - \frac{e^2}{16\pi^2}\frac{3}{2c_Z^2}\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{M_Z^2}\right). \tag{15}$$

We thus replace the bare parameters L_{10} and β_1 with the scale dependent ones above. As a check of our answer, it is interesting to note that we would also obtain a finite answer by adding to the results of Appendix B, the one-loop contributions to the self-energies obtained in unitary gauge in the minimal standard model with one Higgs boson in the loop. Equivalently, the expressions in Eq. 15 correspond to the value of L_{10} and β_1 at one-loop in the minimal standard model within our renormalization scheme.[6]

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[4]That is, a finite answer for the physical observables $Z \to \overline{f}f$, not for quantities like the self-energies.

[5] In these expressions we have simply dropped any finite constants arising from the loop calculation. These constants would only be of interest in a complete effective field theory analysis applied to a problem where all unknown constants can be measured and additional predictions made. For example, that is the status of the $\mathcal{O}(p^4)$ χPT theory for low energy strong interactions.

[6]Our result agrees with that of Refs. [5, 18].

Our result for L_{10} at order $1/\Lambda^2$ is then:

$$\frac{\delta\Gamma_f^{L_{10}}}{\Gamma_0(Z \to f\overline{f})} = \frac{e^2}{c_Z^2 s_Z^2}L_{10}^r(\mu)\frac{v^2}{\Lambda^2}\frac{2r_f(l_f + r_f)}{l_f^2 + r_f^2}\frac{c_Z^2}{s_Z^2 - c_Z^2} \qquad (16)$$

Once again we point out that, at this order, the contribution of L_{10} to the LEP observables occurs only through modifications to the self-energies that are proportional to q^2. At this order it is therefore possible to identify the effect of L_{10} with the oblique parameter S or ϵ_3. If we were to compute the effects of L_{10} at one-loop (as we do for the $L_{9L,9R}$), comparison with S would not be appropriate. Bounding L_{10} from existing analyses of S or ϵ_3 is complicated by the fact that the same one-loop definitions must be used. For example, $L_{10}(\mu)$ receives contributions from the standard model Higgs boson that are usually included in the minimal standard model calculation. We will simply associate our definition of $L_{10}(\mu)$ with *new* contributions to S, beyond those coming from the minimal stardard model.[7]

Numerically we find the following 90% confidence level bounds on L_{10} when we take the scale $\Lambda = 2$ TeV:

$$
\begin{aligned}
\Gamma_e &\rightarrow & -1.7 \leq L_{10}^r(M_Z)_{new} \leq 3.3 \\
R_h &\rightarrow & -1.5 \leq L_{10}^r(M_Z)_{new} \leq 2.0 \\
\Gamma_Z &\rightarrow & -1.1 \leq L_{10}^r(M_Z)_{new} \leq 1.5
\end{aligned} \qquad (17)
$$

We can also bound the *leading order* effects of L_{10} from Altarelli's latest global fit $\epsilon_3 = (3.4 \pm 1.8) \times 10^{-3}$ [1]. To do this, we subtract the standard model value obtained with $160 \leq M_t \leq 190$ GeV and $65 \leq M_H \leq 1000$ GeV as read from Fig. 8 in Ref. [1]. We obtain the 90% confidence level interval:

$$-0.14 \leq L_{10}^r(M_Z)_{new} \leq 0.86 \qquad (18)$$

We can also compare directly with the result of Langacker $S_{new} = -0.15 \pm 0.25^{-0.08}_{+0.17}$[2], to obtain 90% confidence level limits:

$$-0.46 \leq L_{10}^r(M_Z)_{new} \leq 0.77 \qquad (19)$$

The results Eqs.18, 19 are better than our result Eq. 17 because they correspond to global fits that include all observables.

3.2   Bounds on $L_{1,2}$ at order $1/\Lambda^4$.

The couplings $L_{1,2}$ enter the one-loop calculation of the $Z \to f\overline{f}$ width through four gauge boson couplings as depicted schematically in Figure 1. Our prescription calls for using only the leading non-analytic contribution to the process $Z \to f\overline{f}$. This

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[7]We are being sloppy by not matching our L_{10} at one loop with the precise definitions used to renormalize the standard model at one-loop. This does not matter for our present purpose.
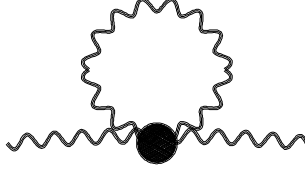
Figure 1: Gauge boson self-energy diagrams involving the couplings $L_{1,2}$.

contribution can be extracted from the coefficient of the pole in $n - 4$. Care must be taken to isolate the poles of ultraviolet origin (which are the only ones that interest us) from those of infrared origin that appear in intermediate steps of the calculation but that cancel as usual when one includes real emission processes as well. We thus use the results of Appendix B with the replacement:

$$\frac{1}{\epsilon} = \frac{2}{4 - n} \rightarrow \log\left(\frac{\mu^2}{M_Z^2}\right) \tag{20}$$

to compute the contributions to the partial widths using Eq. 10.

Since in unitary gauge $L_{1,2}$ modify only the four-gauge boson couplings at the one-loop level, they enter the calculation of the Z partial widths only through the self-energy corrections and Eq. 8. These operators induce a non-zero value for $\Delta\rho \equiv \Pi_{WW}(0)/M_W^2 - \Pi_{ZZ}(0)/M_Z^2$. For the observables we are discussing, this is the *only* effect of $L_{1,2}$. We do not place bounds on them from global fits of the oblique parameter T or ϵ_1, because we have not shown that this is the only effect of $L_{1,2}$ for the other observables that enter the global fits. It is curious to see that even though the operators with L_1 and L_2 violate the custodial $SU(2)_C$ symmetry *only* through the hypercharge coupling, their one-loop effect on the partial Z widths is equivalent to a g^4 contribution to $\Delta\rho$, on the same footing as two-loop electroweak contributions to $\Delta\rho$ in the minimal standard model. The calculation to $\mathcal{O}(1/\Lambda^4)$ can be made finite with the following renormalization of β_1:

$$\beta_1^r(\mu) = \beta_1 + \frac{3}{4}\frac{\alpha^2(1 + c_Z^2)}{s_Z^2 c_Z^4}\left(L_1 + \frac{5}{2}L_2\right)\frac{v^2}{\Lambda^2}\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{M_Z^2}\right). \tag{21}$$

Using our prescription to bound the anomalous couplings, Eq. 20, we obtain for the Z partial widths:

$$\frac{\delta\Gamma_f^{L_{1,2}}}{\Gamma_0(Z \rightarrow f\overline{f})} = -\frac{3}{2}\frac{\alpha^2(1 + c_Z^2)}{s_Z^2 c_Z^4}\left(L_1 + \frac{5}{2}L_2\right)\frac{v^2}{\Lambda^2}\log\left(\frac{\mu^2}{M_Z^2}\right)\left(1 + \frac{2r_f(l_f + r_f)}{l_f^2 + r_f^2}\frac{c_Z^2}{s_Z^2 - c_Z^2}\right). \tag{22}$$

Using $\Lambda = 2$ TeV, and $\mu = 1$ TeV we find 90% confidence level bounds:

$$\Gamma_e \quad \rightarrow \quad -50 \leq L_1 + \frac{5}{2}L_2 \leq 26$$

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$$\Gamma_\nu \quad \rightarrow \quad -28 \le L_1 + \frac{5}{2}L_2 \le 59$$

$$R_h \quad \rightarrow \quad -190 \le L_1 + \frac{5}{2}L_2 \le 130$$

$$\Gamma_Z \quad \rightarrow \quad -36 \le L_1 + \frac{5}{2}L_2 \le 27 \tag{23}$$

Combined, they yield the result:

$$-28 \le L_1 + \frac{5}{2}L_2 \le 26 \tag{24}$$

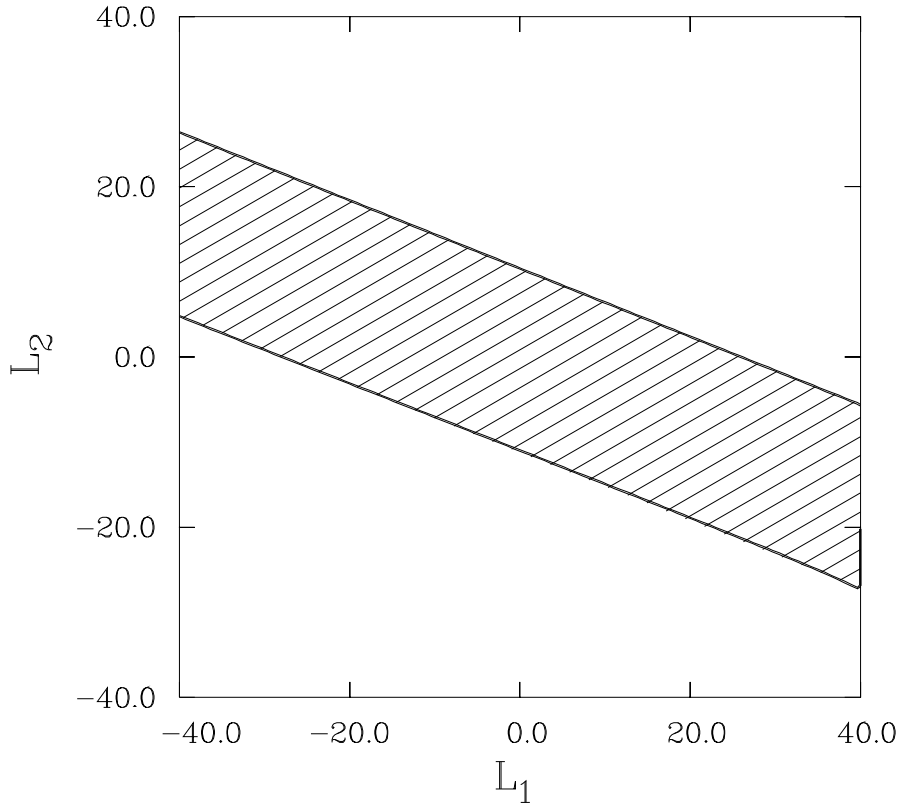shown in Figure 2. As mentioned before, the effect of $L_{1,2}$ in other observables is very

Figure 2: 90% confidence level bounds on $L_{1,2}$ from the $Z \rightarrow f\overline{f}$ partial widths, (Eq. 24). The allowed region is shaded.

different from that of β_1.[8] It is only for the Z partial widths that we can make the $\mathcal{O}(1/\Lambda^4)$ calculation finite with Eq. 21.

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[8] An example are the observables discussed by us in Ref.[19].

3.3 Bounds on $L_{9L,9R}$ at order $1/\Lambda^4$.

The operators with L_{9L} and L_{9R} affect the Z partial widths through Eqs. 7, 8, and 9. We find it convenient to carry out this calculation simultaneously with the one-loop effects of the lowest order effective Lagrangian, Eq. 1, because the form of the three and four gauge boson vertices induced by these two couplings is the same as that arising from Eq. 1. This can be seen from Eqs. 39, 40 in Appendix A. Performing the calculation in this way, we obtain a result that contains terms of order $1/\Lambda^2$ (those independent of $L_{9L,9R}$), terms of order $1/\Lambda^4$ proportional to $L_{9L,9R}$, and terms of order $1/\Lambda^4$ proportional to L_{10} and β_1.

As mentioned before, we keep these terms together to check our answer by taking the limit $L_{9L} = L_{9R} = 0$. This also allows us to cast our answer in terms of g_1^Z, κ_γ and κ_Z which is convenient for comparison with other papers in the literature.

It is amusing to note that the divergences generated by the operators $L_{9L,9R}$ in the one-loop (order $1/\Lambda^4$) calculation of the $Z \to \overline{f}f$ widths can all be removed by the following renormalization of the couplings in Eq. 3 (in the $M_t = 0$ limit):

$$
\beta_1^r(\mu) = \beta_1 - \frac{\alpha}{\pi}\frac{e^2}{96 s_Z^4 c_Z^4}\frac{v^2}{\Lambda^2}\Big[c_Z^2(1-20s_Z^2)L_{9L} + s_z^2(10-29c_Z^2)L_{9R}\Big]\Big(\frac{1}{\epsilon}+\log\frac{\mu^2}{M_Z^2}\Big)
$$

$$
L_{10}^r(\mu) = L_{10} - \frac{\alpha}{\pi}\frac{1}{96 s_Z^2 c_Z^2}\Big[(1-24c_Z^2)L_{9L} + (32c_Z^2-1)L_{9R}\Big]\Big(\frac{1}{\epsilon}+\log\frac{\mu^2}{M_Z^2}\Big) \qquad (25)
$$

This proves our assertion that our calculation to order $\mathcal{O}(1/\Lambda^4)$ can be made finite by suitable renormalizations of the parameters in Eq. 3. However, we do not expect this result to be true in general. That is, we expect that a calculation of the one-loop contributions of the operators in Eq. 4 to other observables will require counterterms of order $1/\Lambda^4$. Thus, Eq. 25 does *not* mean that we can place bounds on $L_{9L,9R}$ from global fits to the parameters S and T. Without performing a complete analysis of the effective Lagrangian at order $1/\Lambda^4$ it is not possible to identify the renormalized parameters of Eq. 25 with the ones corresponding to S and T that are used for global fits.

Combining all the results of Appendix B into Eq. 10, and keeping only terms linear in $L_{9L,9R}$, we find after using Eq. 15, and our prescription Eq. 20:

$$
\frac{\delta\Gamma^{L_9}}{\Gamma_0} = \frac{\alpha^2}{24}\frac{1}{c_Z^4 s_Z^4}\frac{v^2}{\Lambda^2}\log\Big(\frac{\mu^2}{M_Z^2}\Big)
$$
$$
\Bigg\{\Big[L_{9L}(1-24c_Z^2) + L_{9R}(-1+32c_Z^2)\Big]\frac{2r_f(l_f+r_f)}{l_f^2+r_f^2}\frac{c_Z^2}{s_Z^2-c_Z^2}
$$
$$
+2\Big[L_{9L}c_Z^2(1-20s_Z^2) + L_{9R}s_Z^2(10-29c_Z^2)\Big]\Big(1+\frac{2r_f(l_f+r_f)}{l_f^2+r_f^2}\frac{c_Z^2}{s_Z^2-c_Z^2}\Big)\Bigg\}
$$
$$
+\frac{\alpha^2}{12}\log\Big(\frac{\mu^2}{M_Z^2}\Big)\frac{1+2c_Z^2}{c_Z^4 s_Z^4(l_b^2+r_b^2)}\frac{v^2}{\Lambda^2}\frac{M_t^2}{M_Z^2}\Big[L_{9R}s_Z^2 - 7L_{9L}c_Z^2\Big]\delta_{fb} \qquad (26)
$$

The last term in Eq. 26 corresponds to the non-universal corrections proportional to M_t^2 that are relevant only for the decay $Z \to \overline{b}b$.

Using, as before, $\Lambda = 2$ TeV, $\mu = 1$ TeV we find 90% confidence level bounds:

$$
\begin{aligned}
\Gamma_e &\rightarrow & -92 \le L_{9L} + 0.22 L_{9R} \le 47 \\
\Gamma_\nu &\rightarrow & -79 \le L_{9L} + 1.02 L_{9R} \le 170 \\
R_h &\rightarrow & -22 \le L_{9L} - 0.17 L_{9R} \le 16 \\
\Gamma_Z &\rightarrow & -22 \le L_{9L} - 0.04 L_{9R} \le 17
\end{aligned}
\tag{27}
$$

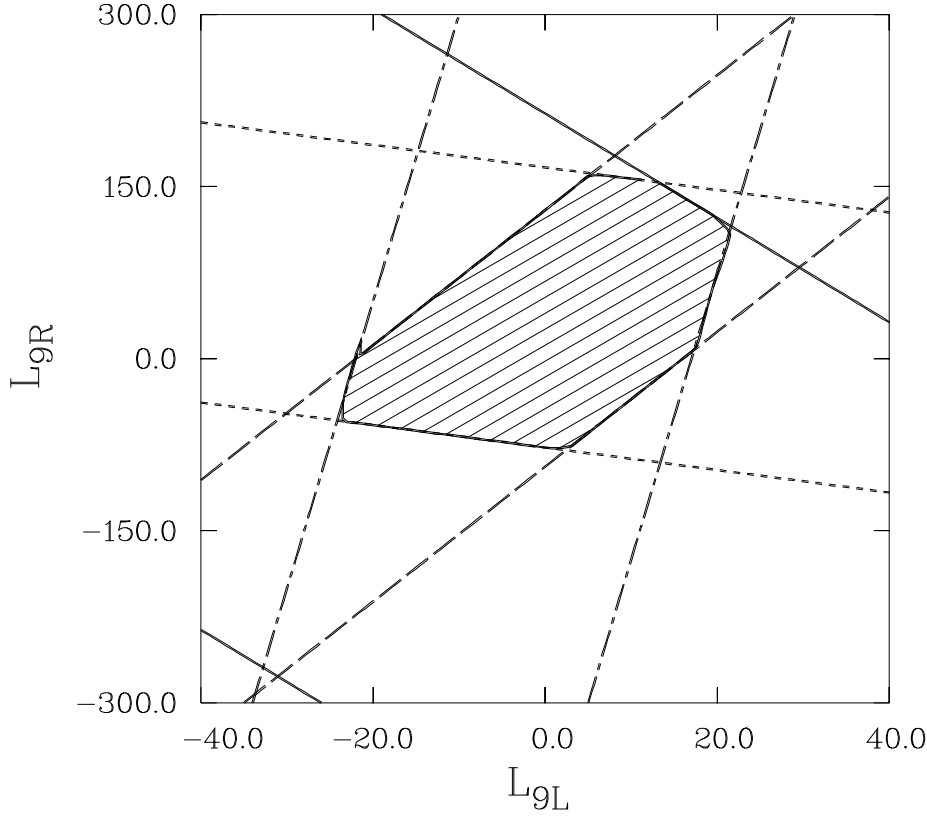We show these inequalities in Figure 3. If we bound one coupling at a time we can

Figure 3: 90% confidence level bounds on $L_{9L,9R}$ from the $Z \rightarrow f\overline{f}$ partial widths, (Eq. 27). The allowed region is shaded. The solid, dotted, dashed, and dot-dashed lines are the bounds from Γ_e, Γ_ν, R_h, and Γ_Z, respectively.

read from Figure 3 that:

$$
\begin{aligned}
-22 \le\ & L_{9L} & \le 16 \\
-77 \le\ & L_{9R} & \le 94
\end{aligned}
\tag{28}
$$

In a vector like model with $L_{9L} = L_{9R}$ we have the 90% confidence level bound:

$$-22 < L_{9L} = L_{9R} < 18. \tag{29}$$

We can relate our couplings of Eq. 4 to the conventional g_1^Z, κ_γ and κ_Z by identifying our unitary gauge three gauge boson couplings with the conventional parameterization of Ref. [21] as we do in Appendix A. However, we must emphasize that there is no unique correspondence between the two. Our framework assumes, for example, $SU(2)_L \times U(1)_Y$ gauge invariance and this results in specific relations between the three and four gauge boson couplings that are different from those of Ref. [22] which assumes only electromagnetic gauge invariance. Furthermore, if one starts with the conventional parameterization of the three-gauge-boson coupling and imposes $SU(2)_L \times U(1)_Y$ gauge invariance one does not generate any additional two-gauge-boson couplings. It is interesting to point out that within our formalism there are only two independent couplings that contribute to three-gauge-boson couplings ($L_{9L,9R}$) but not to two-gauge-boson couplings (as L_{10} does). From this it follows that the equations for g_1^Z, κ_γ and κ_Z in terms of $L_{9L,9R,10}$ are not independent. In fact, within our framework we have:

$$\kappa_Z = g_1^Z + \frac{s_Z^2}{c_Z^2}\left(1 - \kappa_\gamma\right). \tag{30}$$

The same result holds in the formalism of Ref. [23].

For the sake of comparison with the literature we translate the bounds on L_{9L} and L_{9R} into bounds on Δg_1^Z, $\Delta\kappa_Z$ and $\Delta\kappa_\gamma$. For this exercise we set $L_{10} = 0$. We use $L_{9R} = 0$ to obtain the bound on Δg_1^Z. We then solve for L_{9L} and L_{9R} in terms of $\Delta\kappa_Z$ and $\Delta\kappa_\gamma$, and bound each one of these assuming the other one is zero. We obtain the 90% confidence level intervals:

$$
\begin{aligned}
-0.08 < & \quad \Delta g_1^Z \quad < 0.1 \\
-0.3 < & \quad \Delta\kappa_Z \quad < 0.3 \\
-0.3 < & \quad \Delta\kappa_\gamma \quad < 0.4.
\end{aligned}
\tag{31}
$$

Similarly, if there is a non-zero L_{10}, these couplings receive contributions from it. Setting $L_{9L,9R} = 0$, we find from Eq. 17 the bounds: $-0.004 < \Delta g_1^Z < 0.005$, $-0.003 < \Delta\kappa_Z < 0.004$, and $-0.009 < \Delta\kappa_\gamma < 0.007$. These bounds are stronger by a factor of about 20, just as the bounds on L_{10}, Eq. 17 are stronger by about a factor of 20 than the bounds on $L_{9L,9R}$, Eq. 28. However, these really are bounds on the oblique corrections introduced by L_{10} (which also contributes to three gauge boson couplings due to $SU(2)_L \times U(1)_Y$ gauge invariance). It is perhaps more relevant to consider the couplings of operators without tree-level self-energy corrections. This results in Eq. 31.

3.4 Effects proportional to M_t^2

As can be seen from Eq. 26, the $Z \rightarrow \bar{b}b$ partial width receives non-universal contributions proportional to M_t^2. Within our renormalization scheme, the effects that correspond to the minimal standard model do not occur. Our result corresponds entirely to a new physics contribution of order $1/\Lambda^4$ proportional to $L_{9L,9R}$. These effects have already been included to some extent in the previous section when we compared the hadronic and total widths of the Z boson with their experimental values. In this section we isolate the effect of the M_t^2 terms and concentrate on the $Z \rightarrow \bar{b}b$ width. Keeping the leading non-analytic contribution, as usual, we find:

$$\frac{\Gamma(Z \rightarrow \bar{b}b)}{\Gamma_0} - 1 = \frac{\alpha^2}{12} \frac{1 + 2c_Z^2}{c_Z^4 s_Z^4 (l_b^2 + r_b^2)} \frac{v^2}{\Lambda^2} \frac{M_t^2}{M_Z^2} \Big[L_{9R} s_Z^2 - 7 L_{9L} c_Z^2 \Big] \log\Big(\frac{\mu^2}{M_Z^2}\Big) \qquad (32)$$

We use as before $\mu = 1$ TeV, and we neglect the contributions to the $Z \rightarrow \bar{b}b$ width that are not proportional to M_t^2. We can then place bounds on the anomalous couplings by comparing with Langacker's result $\delta_{bb}^{new} = 0.022 \pm 0.011$ for $M_t = 175 \pm 16$ GeV [11]. Bounding the couplings one at a time we find the 90% confidence level intervals:

$$\begin{aligned} -50 \leq \quad L_{9L} \quad &\leq -4 \\ 90 \leq \quad L_{9R} \quad &\leq 1200. \end{aligned} \qquad (33)$$

Once again we find that there is much more sensitivity to L_{9L} than to L_{9R}. The fact that the $Z \rightarrow \bar{b}b$ vertex places asymmetric bounds on the couplings is, of course, due to the present inconsistency between the measured value and the minimal standard model result. Clearly, the implication that the couplings $L_{9L,9R}$ have a definite sign cannot be taken seriously. A better way to read Eq. 33 is thus: $|L_{9L}| \leq 50$ and $|L_{9R}| \leq 1200$.

4 Discussion

Several studies that bound these "anomalous couplings" using the LEP observables can be found in the literature. Our present study differs from those in two ways: we have included bounds on some couplings that have not been previously considered, $L_{1,2}$ and we discuss the other couplings, $L_{9L,9R}$ within an $SU(2)_C \times U(1)_Y$ gauge invariant formalism. We now discuss specific differences with some of the papers found in the literature.

The authors of Ref. [24] obtain their bounds by regularizing the one-loop integrals in n dimensions, isolating the poles in $n - 2$ and identifying these with quadratic divergences. This differs from our approach where we keep only the (finite) terms proportional to the logarithm of the renormalization scale $\log \mu$. To find bounds, the authors of Ref. [24] replace the poles in $n-2$ with factors of Λ^2/M_W^2. We believe that this leads to the artificially tight constraints [25] on the anomalous couplings quoted in

Ref. [24] (2σ limits): $-9.4\times10^{-3} \le \beta_2 \le 2.2\times10^{-2}$ and $-1.5\times10^{-2} \le \beta_3 \le 3.9\times10^{-2}$. We translate these into 90% confidence level intervals:

$$
\begin{aligned}
-1.0 \le\ & L_{9R} &\le 2.4 \\
-1.6 \le\ & L_{9L} &\le 4.2
\end{aligned}
\tag{34}
$$

which are an order of magnitude tighter than our bounds. Conceptually, we see the divergences as being absorbed by renormalization of other anomalous couplings. As shown in this paper, the calculation of the $Z \to \overline{f}f$ can be rendered finite at order $\mathcal{O}(1/\Lambda^4)$ by renormalization of β_1 and L_{10}. Thus, the bounds obtained by Ref. [24], Eq. 34, are really bounds on β_1 and L_{10}. They embody the naturalness assumption that all the coefficients that appear in the effective Lagrangian at a given order are of the same size. Our formalism effectively allows $L_{9L,9R}$ to be different from L_{10}.

The authors of Ref. [22] do not require that their effective Lagrangian be $SU(2)_L \times U(1)_Y$ gauge invariant, and instead they are satisfied with electromagnetic gauge invariance. At the technical level this means that we differ in the four gauge boson vertices associated with the anomalous couplings we study. It also means that we consider different operators. In terms of the conventional anomalous three gauge boson couplings, these authors quote 1σ results $\Delta g_1^Z = -0.040 \pm 0.046$, $\Delta\kappa_\gamma = 0.056 \pm 0.056$, and $\Delta\kappa_Z = 0.004 \pm 0.042$. These constraints are tighter than what we obtain from the contribution of $L_{9L,9R}$ to Δg_1^Z, $\Delta\kappa_\gamma$ and $\Delta\kappa_Z$, Eq. 31; they are weaker than what we obtain from the contribution of L_{10}.

The authors of Ref. [23] require their effective Lagrangian to be $SU(2)_L \times U(1)_Y$ gauge invariant, but they implement the symmetry breaking linearly, with a Higgs boson field. The resulting power counting is thus different from ours, as are the anomalous coupling constants. Their study would be appropriate for a scenario in which the symmetry breaking sector contains a relatively light Higgs boson. Their anomalous couplings would parameterize the effects of the new physics not directly attributable to the Higgs particle. Nevertheless, we can roughly compare our results to theirs by using their bounds for the heavy Higgs case (case (d) in Figure 3 of Ref. [23]). To obtain their bounds they consider the case where their couplings $f_B = f_W$ and $f_{WB} = 0$ which corresponds to our $L_{10} = 0$, and $L_{9L} = L_{9R}$. For $M_t = 170$ GeV they find the following 90% confidence level interval $-0.05 \le \Delta\kappa_\gamma \le 0.12$, which we translate into:

$$
-7.8 \le L_{9L} = L_{9R} \le 18.8
\tag{35}
$$

This compares well with our bound

$$
-22 \le L_{9L} = L_{9R} \le 18
\tag{36}
$$

Finally, if we look *only* at those corrections that are proportional to M_t^2/M_W^2 and that would dominate in the $M_t \to \infty$ limit, we find that they only occur in the $Z \to \overline{b}b$ vertex. This means that they can be studied in terms of the parameter ϵ_b of Ref. [4] or δ_{bb} of Ref. [2]. Converting our result of Eq. 33 to the usual anomalous couplings and

recalling that only two of them are independent at this order, we find, for example:

$$-0.28 \leq \quad \Delta g_1^Z \quad \leq -0.03$$
$$0.6 \leq \quad \Delta \kappa_\gamma \quad \leq 5.2 \tag{37}$$

This result is very similar to that obtained in Ref. [26].

We now compare our results[9] with bounds that future colliders are expected to place on the anomalous couplings. In Fig. 4, we compare our 95% confidence level
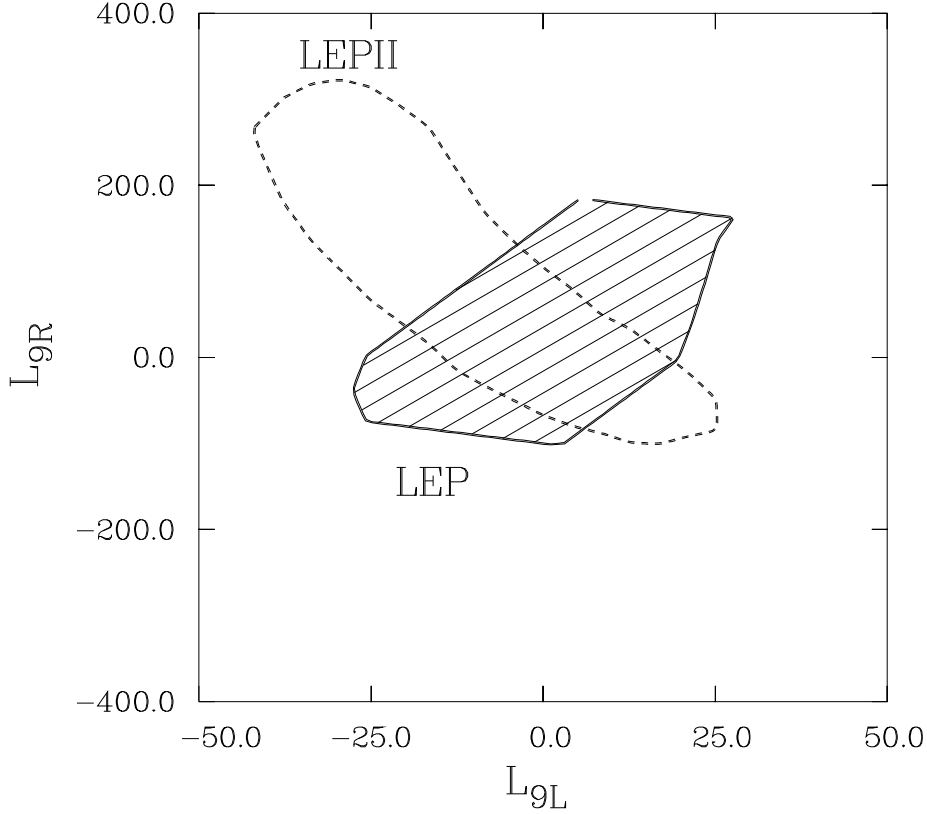

Figure 4: Comparison of the 95% confidence level bounds from the Z partial widths (shaded region) with that obtainable at LEPII with $\sqrt{s} = 196 \ GeV$ and $\int \mathcal{L} = 500 \ pb^{-1}$, (dotted contour) [8].

bounds on L_{9L} and L_{9R} with those which can be obtained at LEPII with $\sqrt{s} =$

---

[9] Our normalization of the L_i is different from that of Ref. [8, 19, 20]. We have translated their results into our notation.

16

196 GeV and an integrated luminosity of 500 pb^{-1} [8, 27]. We find that LEP and LEPII are sensitive to slightly different regions of the L_{9L} and L_{9R} parameter space, with the bounds from the two machines being of the same order of magnitude. The authors of Ref. [20] find that the LHC would place bounds of order $L_{9L} < \mathcal{O}(30)$ and a factor of two or three worse for L_{9R}. We find, Eq. 28, that precision LEP measurements already provide constraints at that level. We again emphasize our caveat that the bounds from LEP rely on naturalness arguments and are no substitute for measurements in future colliders.

The limits presented here on the four point couplings L_1 and L_2 are the first available for these couplings. They will be measured directly at the LHC. Assuming a coupling is observable if it induces a 50% change in the high momentum integrated cross section, Ref. [19] estimated that the LHC will be sensitive to $\mid L_1, L_2 \mid \sim \mathcal{O}(1)$, which is considerably stronger that the bound obtained from the Z partial widths.

5   Conclusions

We have used an effective field theory formalism to place bounds on some non-standard model gauge boson couplings. We have assumed that the electroweak interactions are an $SU(2)_L \times U(1)_Y$ gauge theory with an unknown, but strongly interacting, scalar sector responsible for spontaneous symmetry breaking. Computing the leading contribution of each operator, and allowing only one non-zero coefficient at a time, our 90 % confidence level bounds are:

$$
\begin{aligned}
-1.1 < \quad & L_{10}^r(M_Z)_{new} \quad < 1.5 \\
-28 < \quad & L_1 \quad < 26 \\
-11 < \quad & L_2 \quad < 11 \\
-22 < \quad & L_{9L} \quad < 16 \\
-77 < \quad & L_{9R} \quad < 94.
\end{aligned}
\tag{38}
$$

Two parameter bounds on (L_1, L_2) and (L_{9L}, L_{9R}) are given in the text. The bounds on L_1, L_2 are the first experimental bounds on these couplings. The bounds on L_{9L} and L_{9R} are of the same order of magnitude as those which will be obtained at LEPII and the LHC.

us with his latest numbers. We thank F. Boudjema for providing us with the data file for the LEPII bounds in Figure 4.

A   Three and Four Gauge Boson Couplings in Unitary Gauge

It has become conventional in the literature to parameterize the three gauge boson vertex VW^+W^- (where $V = Z, \gamma$) in the following way [21]:

$$
\begin{aligned}
\mathcal{L}_{WWV} &= -ie\frac{c_Z}{s_Z}g_1^Z\Big(W_{\mu\nu}^\dagger W^\mu - W_{\mu\nu}W^{\mu\,\dagger}\Big)Z^\nu - ieg_1^\gamma\Big(W_{\mu\nu}^\dagger W^\mu - W_{\mu\nu}W^{\mu\,\dagger}\Big)A^\nu \\
&\quad -ie\frac{c_Z}{s_Z}\kappa_Z W_\mu^\dagger W_\nu Z^{\mu\nu} - ie\kappa_\gamma W_\mu^\dagger W_\nu A^{\mu\nu} \\
&\quad -e\frac{c_Z}{s_Z}g_5^Z\epsilon^{\alpha\beta\mu\nu}\Big(W_\nu^-\partial_\alpha W_\beta^+ - W_\beta^+\partial_\alpha W_\nu^-\Big)Z_\mu \quad .
\end{aligned}
\tag{39}
$$

Terms of the form $(\lambda_V/M_V^2)W_{\rho\mu}^\dagger W_\sigma^\mu V^{\sigma\rho}$ which are often included in the parameterization of the three gauge boson vertex do not appear in our formalism to the order we work.

For calculations to order $1/\Lambda^2$, it is most convenient to diagonalize the gauge-boson self-energies as done in Ref. [15]. This results in expressions for g_1^Z, κ_Z and κ_γ in terms of $L_{9L,9R,10}$ that we presented in Ref. [16]. For the present study, we do not keep the L_{10} or β_1 terms as explained in the text. We thus use:

$$
\begin{aligned}
g_1^Z &= 1 + \frac{e^2}{2c_Z^2 s_Z^2}L_{9L}\frac{v^2}{\Lambda^2} \\
g_1^\gamma &= 1 \\
\kappa_Z &= 1 + \frac{e^2}{2s_Z^2 c_Z^2}\Big(L_{9L}c_Z^2 - L_{9R}s_Z^2\Big)\frac{v^2}{\Lambda^2} \\
\kappa_\gamma &= 1 + \frac{e^2}{2s_Z^2}\Big(L_{9L} + L_{9R}\Big)\frac{v^2}{\Lambda^2}.
\end{aligned}
\tag{40}
$$

The four gauge boson interactions derived from Eqs. 1 and 4 after diagonalization of the gauge boson self-energies can be written as:

$$
\begin{aligned}
\mathcal{L}_{WWV_iV_j} &= C_{ij}\Big(2W^+\cdot W^- V_i\cdot V_j - V_i\cdot W^+ V_j\cdot W^- - V_j\cdot W^+ V_i\cdot W^-\Big) \\
&\quad +\frac{e^4}{s_Z^4}\frac{v^2}{\Lambda^2}\Big[\frac{1}{c_Z^2}\Big(L_1 W^+\cdot W^- Z\cdot Z + L_2 W^+\cdot Z W^-\cdot Z\Big) \\
&\quad +(L_1 + \frac{L_2}{2})(W^+\cdot W^-)^2 + \frac{L_2}{2}W^+\cdot W^+ W^-\cdot W^- \\
&\quad +\frac{1}{4c_Z^4}(L_1 + L_2)(Z\cdot Z)^2\Big]
\end{aligned}
\tag{41}
$$

where $V_i = \gamma, Z$ or $W^{\pm}$ and,

$$
\begin{aligned}
C_{\gamma\gamma} &= -e^2 \\
C_{ZZ} &= -e^2 \frac{c_Z^2}{s_Z^2} (g_1^Z)^2 \\
C_{\gamma Z} &= -e^2 \frac{c_Z}{s_Z} g_1^Z \\
C_{WW} &= -\frac{e^2}{s_Z^2} \left(1 + 2c_Z^2 (g_1^Z - 1) \right).
\end{aligned} \tag{42}
$$

B   One-loop Results

As explained in the text, we will only consider the tree-level effects of L_{10}. This means that for the one-loop calculation to order $1/\Lambda^4$ only $L_{9L,9R}$ appear in Eq. 40. For the calculation to order $1/\Lambda^2$ presented in this paper, we do not use the diagonal basis, but rather obtain our results from the explicit factors of L_{10} and β_1 that appear in the following expressions.

The vector boson self energies can be written in the form:

$$
-i\Pi_{VV}^{\mu\nu}(p^2) = A_{VV}(p^2)g^{\mu\nu} + B(p^2)p^\mu p^\nu. \tag{43}
$$

We regularize in n dimensions and keep only the poles of ultraviolet origin. For the case of fermion loops we treat all fermions as massless except the top-quark. We find:

$$
\begin{aligned}
\frac{A_{\gamma\gamma}(p^2)}{p^2} &= -\frac{\alpha}{4\pi\epsilon} \left\{ \frac{p^2}{3M_W^2} - 1 + \frac{\kappa_\gamma^2}{12} \frac{p^2}{M_W^4}(p^2 - 2M_W^2) + \frac{\kappa_\gamma}{M_W^2}(p^2 - 6M_W^2) \right\} \\
&\quad - \frac{\alpha}{12\pi s_Z^4 \epsilon} \sum_f N_{Cf} r_f^2 + 8\pi\alpha \frac{v^2}{\Lambda^2} L_{10} \\
\frac{A_{\gamma Z}(p^2)}{p^2} &= \frac{\alpha}{4\pi\epsilon} \left(\frac{c_Z}{s_Z} \right) \left\{ g_1^Z (1 - \frac{p^2}{3M_W^2}) - \frac{\kappa_Z \kappa_\gamma p^2}{12M_W^4}(p^2 - 2M_W^2) \right. \\
&\quad \left. - \frac{(g_1^Z \kappa_\gamma + \kappa_Z)}{2M_W^2}(p^2 - 6M_W^2) \right\} \\
&\quad + \frac{\alpha}{24\pi s_Z^3 c_Z \epsilon} \sum_f N_{Cf} r_f \left(r_f + l_f \right) + 4\pi\alpha \frac{c_Z^2 - s_Z^2}{s_Z c_Z} \frac{v^2}{\Lambda^2} L_{10} \\
\frac{A_{ZZ}(p^2)}{p^2} &= \frac{\alpha}{4\pi\epsilon} \left(\frac{c_Z}{s_Z} \right)^2 \left\{ g_1^{Z\,2}(1 - \frac{p^2}{3M_W^2}) - \frac{\kappa_Z^2 p^2}{12M_W^4}(p^2 - 2M_W^2) - \frac{g_1^Z \kappa_Z}{M_W^2}(p^2 - 6M_W^2) \right\} \\
&\quad - \frac{\alpha}{8\pi s_Z^2 c_Z^2 \epsilon} \left[-\frac{3M_t^2}{p^2} + \sum_f \frac{N_{Cf}}{3} \left(r_f^2 + l_f^2 \right) \right] - 2\frac{M_Z^2}{p^2}\beta_1 - 8\pi\alpha \frac{v^2}{\Lambda^2} L_{10} \tag{44}
\end{aligned}
$$

These results can be compared with the unitary gauge results of Degrassi and Sirlin in the standard model limit ($g_1^Z = \kappa_Z = \kappa_\gamma = 1$). When the contribution of the standard model Higgs boson is included, Eq. 44 agrees with Ref. [28]

For the renormalization of G_F, we need $A_{WW}(q^2 = 0)$, the $We\nu$ vertex evaluated at $q^2 = 0$, $\Gamma_{We\nu}$, the box contribution to $\mu \to e\nu\overline{\nu}$, B_{box}, and the charged lepton wavefunction renormalization, Z_f:

$$
\begin{aligned}
A_{WW}(0) &= \frac{3\alpha}{16\pi\epsilon} M_W^2 \left\{ \kappa_\gamma^2 + 2 + 2\kappa_\gamma - \frac{3}{s_Z^2}(1 - 2c_Z^2) \right. \\
&\quad + \left(\frac{1}{s_Z^2}\right) \left[\kappa_Z^2 \left(c_Z^2 + 1 - \frac{1}{c_Z^2} \right) + g_1^{Z\,2} \left(c_Z^2 - 2 - c_Z^4 \right) \right. \\
&\quad \left. \left. + 2\kappa_Z g_1^Z \left(c_Z^2 + 1 \right) - 6c_Z^2 g_1^Z \right] \right\} + \frac{3\alpha}{8\pi s_Z^2 \epsilon} M_t^2 \\
\Gamma_{We\nu}(0) &= \mathcal{A}_0 \frac{3\alpha}{8\pi\epsilon} \left\{ \left(1 + \frac{1}{2}\kappa_\gamma \right) + \frac{c_Z^2}{2s_Z^2} \left[g_1^Z \left(1 - c_Z^2 \right) + \kappa_Z \left(1 - \frac{1}{c_Z^2} \right) \right] \right\} \\
\mathcal{A}_0 &= -\frac{g}{2\sqrt{2}} \overline{e}\gamma^\mu(1 - \gamma_5)\nu\epsilon_\mu^W \\
B_{\text{box}} &= \frac{3\alpha}{16\pi\epsilon} \left(\frac{c_Z^2}{s_Z^2}\right)(1 + c_Z^2) + \frac{\alpha}{4\pi\epsilon} \\
Z_f - 1 &= -\frac{\alpha r_f^2}{16 s_Z^4 \pi\epsilon}
\end{aligned}
\tag{45}
$$

For massless fermions in dimensional regularization there is a cancellation between the ultraviolet and infrared divergent contributions, responsible for the familiar result that their wavefunction renormalization vanishes. We are isolating the ultraviolet divergences only, so we obtain a contribution to the fermion wavefunction renormalization.

Figure 5: Diagrams contributing to δl_f.

The corrections to the $Z(p^2)\overline{f}f$ vertex from the diagrams shown in Fig. 5 (including in this term the wave function renormalization for the external fermion) are:

$$
\begin{aligned}
\delta l_f &= (l_f - r_f) \frac{\alpha}{4\pi\epsilon} \left(\frac{c_Z}{s_Z}\right)^2 \left\{ \kappa_Z \frac{p^2}{M_W^2} \left(\frac{1}{2} + \frac{p^2}{12M_W^2} \right) + g_1^Z \left(\frac{5p^2}{6M_W^2} \right) \right\} \\
&\quad + \frac{\alpha}{16\pi s_Z^2 \epsilon} \frac{M_t^2}{M_W^2} \left[r_t - 4l_t + (\kappa_Z \frac{p^2}{M_Z^2} + 6c_Z^2 g_1^Z) + 3L_b \right] \delta_{fb}
\end{aligned}
\tag{46}
$$

When the wavefunction renormalization is included in the definition of δr_f, we have $\delta r_f = 0$ from the diagrams of Fig. 5.

The Z wavefunction renormalization is given by:

$$
\begin{aligned}
Z_Z - 1 \;=\; & -\frac{\alpha}{8\pi s_Z^2 c_Z^2 \epsilon} \sum_f \frac{N_{Cf}}{3}\Big(r_f^2 + l_f^2\Big) - 8\pi\alpha\frac{v^2}{\Lambda^2}L_{10} \\
& +\frac{\alpha}{4\pi\epsilon}\Big(\frac{c_Z}{s_Z}\Big)^2\Big\{ g_1^{Z\;2}(1-\frac{2}{3c_Z^2}) - \frac{\kappa_Z^2}{12}(\frac{3}{c_Z^4}-\frac{4}{c_Z^2}) - g_1^Z\kappa_Z(\frac{2}{c_Z^2}-6)\Big\} \quad (47)
\end{aligned}
$$

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